Simulating a Quantum Dot Thermometer using Feynman's Vector Model for Two-Level Systems

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***Abstract*—This research aims to demonstrate that Feynman's vector model can be effectively used to simulate a quantum dot thermometer for nanoscale temperature measurement. Combining theoretical quantum principles with computational modeling, we model quantum dots as precessing vectors on a Bloch sphere, linking excited-state and ground-state population dynamics to temperature via the Boltzmann distribution. Photoluminescence intensity gives the primary temperature readout. Simulations confirm the model's accuracy at the nanoscale, demonstrating that Feynman's geometrical approach simplifies complex quantum problems and provides a promising basis for future applications such as targeted drug delivery or diagnostics for quantum computing hardware.**

A challenging frontier in modern physics is measuring temperature at a nanoscale. As the boundaries of physics are pushed to an ever-smaller domain, conventional thermometric techniques become insufficient. This research explores an innovative solution to this challenge by simulating a quantum dot thermometer based on Feynman's vector model for two-level systems, as presented in his 1957 paper "Geometrical Representation of the Schrödinger Equation for Solving Master Problems" [1].

The foundation of this research lies in Feynman's important insight that the complex behaviour of two-level systems can be represented using a simple vector model. This three-dimensional vector called r, in a Bloch sphere, maps the quantum state of a two-level system. The vector points from the center to a location on the sphere's surface. The north pole represents a complete excited state, while the south pole represents a complete ground state. All other points in between correspond to superpositions. This geometric approach by Feynman transforms an abstract quantum mechanical problem into a classic vector problem. This way, phenomena like coherence become as tangible as the precession of a spinning top.

A quantum dot serves as a great example of Feynman's theory. These semiconductors behave like artificial atoms, making them a perfect example of a two level system [3]. When illuminated with light, quantum dots absorb and emit photons at specific wavelengths determined by the structure of their energy levels. Crucially, the relative populations of these energy states follow the Boltzmann distribution, creating a connection between the quantum dot's optical properties and its thermal environment. This connection forms the basis for our thermometer.

The implementation of this concept relies on a few key physical phenomena. First, the vertical component of Feynman's vector, r_3, directly indicates the population of electrons in the excited and ground

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states. Second, the intensity of the light emitted by the quantum dot provides a measurable proxy for this population. Third, by combining these observations with the Boltzmann distribution, we can extract the environmental temperature with remarkable precision. This chain of reasoning transforms what might appear as abstract quantum mechanics into a practical measurement tool.

This research specifically focuses on developing a simulation framework for such a quantum dot thermometer, with potential applications in monitoring nanoscale thermal environments. The ability to measure temperature at this scale could be a great advancement in fields like targeted drug delivery, where precise thermal monitoring is crucial. The quantum dot approach to measuring temperature in a nanoscale offers significant advantages over conventional methods, including minimal invasiveness, high spatial resolution, and the potential for remote readout through optical techniques.

The theoretical foundation of this work is influenced by Feynman's original formulation, which showed how the time evolution of quantum states could be described using classical vector precession equations [2]. This correlation allows us to evade much of the mathematical complexity typically associated with quantum mechanics while retaining all the essential physics. In our simulation, each quantum dot is represented by its own Feynman vector, with the ensemble behavior emerging from the statistical mechanics of these individual systems interacting with a thermal environment.

This research addresses the critical gap in nanoscale temperature measurement by leveraging quantum dots as minimally invasive optical thermometers. While existing techniques struggle with spatial resolution or biological compatibility, Feynman's vector model provides the missing theoretical framework to get precise temperature data from quantum dot photoluminescence. The resulting approach combines fundamental quantum principles with practical applications, enabling thermal monitoring in environments where conventional methods fail, particularly within living systems where temperature regulation is crucial.

Method

A quantum dot is just like an artificial atom. In this case a two level quantum dot is an example of a quantum dot that has two distinctive energy levels: a lower energy level called the ground state and a higher energy level called the excited state. This makes it an excellent example of a two level system described by Feynman. ΔE is the energy gap that separates these two levels.

To move from an abstract quantum description to a geometric one, Feynman's model can be used. He defined the three dimensional vector $\vec{r} = (r_1, r_2, r_3)$ whose components are constructed from probability amplitudes (eq (1,2,3)).

$$r_1 = ab* + ba* \tag{1}$$

$$r_2 = i\,(ab* - ba*) \tag{2}$$

$$r_3 = aa* - bb* \tag{3}$$

This vector lives on the surface of a unit sphere (the Bloch sphere). All of its three components have a direct physical interpretation. The vertical component r_3 is the population difference. It is defined as the probability of being in the excited state minus the probability of being in the ground state (eq.4).

$$r_3 = |a|^2 - |b|^2 \tag{4}$$

When $r_3 = +1$, the quantum dot is completely in the excited state, when $r_3 = -1$, the quantum dot is completely in the ground state. Additionally, a value of $r_3 = 0$ indicates an equal superposition of both states. The horizontal components, r_1 and r_2, represent the "coherence" of the system. They describe the phase relationship between the states and are related to the system's electric dipole moment, which dictates its ability to emit or absorb radiation.

The time-evolution of the vector r under an external perturbation is shown by the equation 5:

$$\frac{d\vec{r}}{dt} = \vec{\omega} x \vec{r} \tag{5}$$

In this classical equation, ω is a vector that represents the external perturbation (e.g., an oscillating electric field from a laser). This equation allows us to simulate the quantum dynamics of the dot using straightforward vector calculus.

Once thermal equilibrium is reached, the external perturbation is removed and the system only interacts with its thermal environment. The coherence components average to zero for an ensemble of dots, and only the population difference r_3 remains significant. The average value of r_3 for an ensemble of quantum dots can be connected to the Boltzmann distribution to find the temperature.

The Boltzmann distribution states the ratio of the number of quantum dots in the excited state (N_e) to the number of quantum dots in the ground state (N_g) at a temperature T. This is given by [4]:

$$\frac{N_e}{N_g} = e^{\frac{-\Delta E}{kT}} \tag{6}$$

In equation 6, ΔE is the energy difference between the two states, k is the Boltzmann constant, and T is the temperature in Kelvin.

The probabilities can be expressed in terms of r_3. Recall from before that $|a|^2 = \frac{N_e}{N_{total}}$ and $|b|^2 = N_g - Ntotal$, and $r_3 = |a|^2 - |b|^2$. Using the identity $|a|^2 + |b|^2 = 1$, we can solve for the excited state probability. Substituting these into the Boltzmann equation gives the crucial link between the measurable Feynman vector component and temperature (eq.7):

$$\frac{N_e}{N_g} = \frac{1 + r_3}{1 - r_3} = e^{\frac{-\Delta E}{kT}} \tag{7}$$

The vector r_3 is not measured directly. Instead, the photoluminescence (PL) intensity of the light emitted by the quantum dot is measured. A weak laser excites the ensemble of quantum dots, and a spectrometer measures the intensity I_{PL} of the light they emit upon returning to the ground state.

This intensity is directly proportional to the number of quantum dots in the excited state. The maximum possible intensity, I_{max}, occurs when all dots are excited $(r_3 = 1)$. Hence, the normalized intensity gives a direct measure of the excited state population (eq.8):

$$\frac{I_{PL}}{I_{max}} = |a|^2 = \frac{1 + r_3}{2} \tag{8}$$

This can be rearranged to solve for r_3 from the intensity measurements:

$$r_3 = 2(\frac{I_{PL}}{I_{max}}) - 1 \tag{9}$$

The found value of r_3 can then be inserted into the derived Boltzmann relation to calculate the temperature T:

$$\frac{1 + (2(\frac{I_{PL}}{I_{max}}) - 1)}{1 - (2(\frac{I_{PL}}{I_{max}}) - 1)} = e^{\frac{-\Delta E}{kT}} \tag{10}$$

The equation 10 forms the core of the quantum dot thermometer, transforming a measurement of light intensity into a precise reading of nanoscale temperature.

Simulation

The simulation of the quantum dot thermometer's behavior was performed using a Python script. The simulation is based on Feynman's vector model, incorporating both coherent precision due to the intrinsic energy gap and incoherent relaxation due to interactions with a thermal environment.

For the means of the example simulation, the following parameters were used:

- The intrinsic energy gap of the quantum dot was set to a frequency of 15 GHz. This value defines the magnitude of the ω_3 component of the Hamiltonian vector.
- A relaxation rate of $10^9 s^{-1}$ was used. This parameter represents the coupling between the quantum dot and its thermal environment; it dictates the timescale over which the dot forgets its initial state and settles into thermal equilibrium.

The time evolution of the state vector was computed by numerically integrating the damped precession equation using the solve_ivp function from the scipy.integrate library [5]. The simulation was run for a duration sufficient to allow the state vector to approach its equilibrium value, typically several times the inverse of the relaxation rate $(1/\Gamma)$.

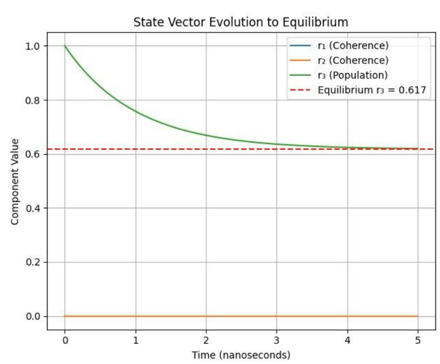

Figure 1. The trajectory of the components of the r vector as time passes next to the equilibrium population vector, at $T = 0.5$ K.

Following the time evolution simulation, a calibration curve was generated by calculating the equilibrium population difference (r_3) across a range of temperatures (from 0.01 K to 5.0 K) using the Boltzmann distribution formula.

Finally, the process of temperature measurement was simulated by selecting a hypothetical measured r_3 value and using linear interpolation on the generated calibration curve to infer the corresponding temperature.

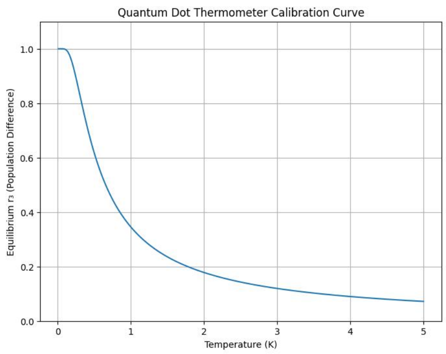

Figure 2. The trajectory of the population vector on the Bloch sphere as it approaches thermal equilibrium.

Discussion

The findings of this simulation provide insight into how temperature can be measured on a nanoscale using quantum dots by incorporating the ideas of Feynman's vector model for two-level systems. By treating each quantum dot as a Bloch vector precessing under external influence, we were able to simulate an ensemble average value for the r vector at thermal equilibrium. By substituting this value into the equation derived from the Boltzmann distribution, we were able to accurately find the temperature value. This application validates the assumption that the abstract quantum state of a two-level system can be geometrically represented for practical measurement. The key interpretation is that Feynman's model acts as a powerful simplifying bridge, translating a complex quantum mechanical problem into a classical vector precession problem, just like a spinning top in a gravitational field.

While the use of quantum dots and the Boltzmann distribution for temperature sensing is evident in existing literature, this researcher's approach differs in that it utilizes Feynman's geometrical representation as a key concept. This approach not only provides a much better understanding of the system's dynamics but also offers a simulation methodology that avoids the computational heaviness of full wavefunction evolution. The primary use of this work is its potential as an educational and prototyping tool, providing a clear pathway from theory to application. The next step in this study would be to make this quantum dot thermometer suitable for work inside a human. This would be of great help for measuring the temperature of nanobots in cases like remote drug delivery inside the human body.

Conclusion

In this research, a successful simulation of a quantum dot thermometer has been created. Connecting this quantum thermometer with Feynman's geometric model for two-level systems takes the complex and abstract nature behind this quantum problem and transforms it into a much more tangible one, making it easier to understand and visualize. A promising next step is to demonstrate the quantum dot thermometer's application in areas such as biomedical sensing within the human body and diagnostics for quantum computing hardware.

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