Quantum Approximate Optimization Algorithm for Large-Scale Portfolio Selection with Multi-Factor Constraints

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***Abstract*—Large-scale portfolio optimization is hard because investors must balance risk, return, diversification, and liquidity across many assets with complex constraints. Classical methods, such as quadratic programming or metaheuristics, often become slow or less effective as the number of assets and factors grows. In this project, I explore the use of the Quantum Approximate Optimization Algorithm (QAOA) as a hybrid quantum–classical approach to portfolio selection. The framework encodes portfolio decisions as qubits and represents financial objectives—such as mean–variance trade-offs or risk-adjusted return—inside a parameterized quantum circuit. Classical optimization is then used to tune the circuit parameters so that the quantum system converges toward low-risk, high-return portfolios. Multi-factor constraints, including sector limits, liquidity rules, ESG scores, and macroeconomic risk factors, are integrated into the Hamiltonian as penalty terms. A qualitative comparison with classical solvers (e.g., mixed-integer programming and simulated annealing) suggests that QAOA can scale more gracefully in high-dimensional settings and handle complex constraint structures. Overall, the project shows how quantum hybrid optimization could support future portfolio management, linking theoretical quantum algorithms with practical, sustainability-aware investment strategies.**

Portfolio optimization is a core problem in quantitative finance. Classical models, such as Markowitz mean–variance theory, search for asset allocations that minimize risk for a given expected return (Alonso, Camarena, & Guerrero, 2025). In practice, however, modern portfolios must respect many extra conditions: sector caps, liquidity rules, regulatory limits, and sustainability preferences like ESG scores (Olsson & Akkaya, 2025). These additions make the optimization problem high-dimensional and sometimes non-convex, which is challenging for standard methods.

Quantum computing introduces a new way to tackle such problems by using qubits, superposition, and entanglement to explore solution spaces more efficiently. The Quantum Approximate Optimization Algorithm (QAOA) is a hybrid quantum–classical algorithm designed for tough combinatorial optimization tasks (Prasad, Chatrati, & Masthan, 2025). By encoding portfolio decisions into quantum states, QAOA can,

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in principle, evaluate many portfolio configurations at once (Alkhalifa et al., 2025).

This project applies QAOA to large-scale, multi-factor portfolio selection. The aim is to see how a quantum-based framework could handle realistic constraints—including ESG and macroeconomic factors—while remaining compatible with current Noisy Intermediate-Scale Quantum (NISQ) hardware (Liu et al., 2025; Lamichhane & Rawat, 2025).

Methods

Problem Formulation and Multi-Factor Constraints

The portfolio selection problem is formulated as a binary or discrete optimization task:

- Decision variables represent whether an asset is included and, in extended versions, its approximate weight.
- The objective combines expected return and risk (e.g., variance, downside risk, or a Sharpe-like measure).
- Constraints include diversification requirements, maximum exposure to sectors, liquidity thresholds, ESG scores, and macro risk factors.

These elements are translated into a cost function that penalizes portfolios with poor performance or constraint violations.

QAOA Encoding and Quantum Circuit Design

The cost function is mapped into a Hamiltonian suitable for QAOA:

- Each asset or decision variable becomes one or more qubits.
- Objective and constraint terms are encoded as energy contributions in the Hamiltonian.
- QAOA alternates between two types of quantum gates: one related to the cost Hamiltonian and one to a mixing Hamiltonian.

A classical optimizer (e.g., gradient-free search) tunes the circuit parameters to minimize the expected cost. This hybrid loop continues until the algorithm converges to a set of promising portfolios.

Comparison with Classical Benchmarks

The QAOA-based framework is qualitatively compared against:

- Classical mixed-integer or quadratic programming solvers,
- Heuristic or metaheuristic methods such as simulated annealing or genetic algorithms (Alkhalifa et al., 2025).

The comparison uses insights from the literature on quantum optimization in other domains (Liu et al., 2025; Lamichhane & Rawat, 2025) to discuss scaling, flexibility, and suitability for complex constraints.

Results

Expected Optimization Performance

Based on the models and reviewed studies:

- Scalability: QAOA is expected to handle larger universes of assets more efficiently than some classical heuristics, especially when constraints are numerous and interdependent.
- Constraint handling: Encoding constraints directly into the Hamiltonian as penalty terms allows the algorithm to search within a structured solution space aligned with real investment rules.
- Solution quality: QAOA aims for near-optimal portfolios, trading exact optimality for significant gains in speed and flexibility—an acceptable trade-off in many real-world settings.

ESG and Multi-Factor Integration

By including ESG indicators and macroeconomic risk factors as separate terms in the Hamiltonian, the framework:

- Encourages portfolios that meet minimum sustainability scores,
- Limits exposure to sectors or regions with higher systemic risk,
- Balances financial performance with long-term resilience and responsibility (Cuomo & Foroudi, 2025; Chamola et al., 2025).

This shows how quantum optimization can support sustainable finance goals, not just pure return maximization.

Discussion

Implications for Quantum Finance

The study suggests that QAOA can act as a bridge between theoretical quantum algorithms and practical portfolio management:

- It offers a structured way to include many realistic constraints,

- It fits current NISQ hardware, allowing experimentation today,
- It can evolve as quantum devices become more powerful.

This aligns with broader trends where quantum methods are being explored for business, energy systems, and financial decision-making (Liu et al., 2025; Cuomo & Foroudi, 2025).

Limitations and Future Work

There are important limitations:

- Hardware noise and limited qubit counts restrict the size of portfolios that can be tested today.
- Encoding continuous weights remains challenging and may require approximations or hybrid schemes.
- Real financial data and stress conditions must be used in future experiments to validate the approach.

Future work could focus on improved encodings, error mitigation, and real-world case studies with institutional partners.

Conclusion

This project presents a QAOA-based framework for large-scale portfolio selection with multi-factor constraints. By encoding assets, risk–return objectives, and ESG/macro constraints into a quantum Hamiltonian and using a hybrid quantum–classical loop, it outlines how quantum optimization could support next-generation portfolio management.

While still conceptual, the approach shows how quantum finance might move from theory to practice, enabling more flexible, scalable, and sustainability-aware investment strategies as quantum hardware continues to develop.

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